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The shear viscosity of the non-commutative plasma

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Abstract: We compute the shear viscosity of the non-commutative N=4 super Yang-Mills quantum field theory at strong coupling using the dual supergravity background. Special interest derives from the fact that the background presents an intrinsic anisotropy in space through the distinction of commutative and non-commutative directions. Despite this anisotropy the analysis exhibits the ubiquitous result $\eta/s = 1/4\pi$ for two different shear channels. In order to derive this result, we show that the boundary energy momentum tensor must couple to the open string metric. As a byproduct we compute the renormalised holographic energy momentum tensor and show that it coincides with one in the commutative theory.

Keywords: Non-Commutative Geometry, AdS-CFT Correspondence, Thermal Field Theory.

Contents

1. Introduction

One of the most interesting developments and applications of string theory in the recent years has been the study of the properties of strongly coupled gauge theories at high temperatures via the AdS/CFT duality [1]. At sufficiently high temperatures gauge theories go over into a plasma phase. According do the AdS/CFT dictionary the plasma phase is represented in the holographic dual as a gravitational background containing a black hole [2].

Earlier applications of asymptotically AdS black holes to the study of strongly coupled gauge theories have focused on the thermodynamics of the system, typically studying Hawking temperature, entropy, free energy and phase transitions, which can be studied using the Euclidean section of the black hole metrics.

The recent interest has however derived from the study of non-equilibrium dynamics and here it is essential to use the real time Lorentzian background [3]. One of the main motivations is the experimental progress in the study of the Quark-Gluon plasma at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven. It turned out that the state of matter created at the heavy ion collisions at RHIC is most likely to be understood as a strongly coupled Quark Gluon plasma (for a review see [4]). This poses great problems for the theory since usual perturbative field theory techniques are not applicable and lattice simulations are mostly confined to the equilibrium regime. Therefore the AdS/CFT duality has emerged as a useful tool for studying the properties of the plasma phase on non-abelian gauge theories at strong coupling.

One of the most impressive results in this line of investigation has been the calculation of the shear viscosity in holographic gauge theories [5]. There are two extremely interesting aspects of these kind of calculations. One is that the actual numerical value turns out to be consistent with experiments (see e.g. [6]). The other, more theoretical one, is the fact that all calculations that have been done so far in different holographic theories have always produced the same result for the ration of the shear viscosity over the entropy

$$\frac{\eta}{s} = \frac{1}{4\pi} \,. \tag{1.1}$$

This gave rise to the conjecture that the value is universal and represents a lower bound for all physical systems [7].[1] This universality was first suggested through a case by case investigation, while attempts at a general proof have always faced some restrictions over the class of metrics involved [9].

As system under consideration we have chosen the holographic dual to the non-commutative $\mathcal{N} = 4$ gauge theory [10, 11]. We will see that understanding holography in this theory is by itself an interesting endeavour. In view of the calculations of the shear viscosity additional interest derives from the fact that the non-commutative theory is anisotropic in space. We will consider the theory with two space coordinates (y, z) being non-commutative and leave (t, x) usual commutative spacetime coordinates. Now we remember that the viscosity is actually a fourth rank tensor relating the gradients in the local fluid velocity to the dissipative part of the stress tensor

$$T^D_{ij} = \eta_{ij,kl} \frac{\partial u_k}{\partial x_l} \,. \tag{1.2}$$

Assuming the usual SO(3) rotational invariance leaves only two independent tensor structures, corresponding to shear viscosity η and bulk viscosity ζ. In the anisotropic, non-commutative theory we have however only an SO(2) symmetry and this gives rise to a much richer tensor structure. We will be interested only in the shear part, i.e. the part that determines the diffusion of momentum into transverse directions. More precisely we have to distinguish between three possible shear viscosity coefficients corresponding to either momentum in the commutative direction diffusing into a non-commutative direction, momentum in a non-commutative direction diffusing into a commutative direction and momentum in a non-commutative direction diffusing into a non-commutative direction.[2]

On a technical level the shear viscosities can be calculated in various different ways, either by searching for a diffusion pole in the retarded two point function of the momentum density operators or by Green-Kubo type formulas using the zero-momentum and zero frequency limit of the imaginary part of the retarded equilibrium Greens function of traceless components of the stress tensor.

We therefore need to understand first how the field theory stress tensor can be obtained from the holographic dual. In holographic duals the stress tensor is usually generated by a fluctuation of the metric components. In the non-commutative theory it is important to

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[1] See however recent attempts to construct "gedanken" counterexamples in [8].

[2] Anisotropic shear viscosities are well-known in the theory of liquid crystals where there is a director field indicating a preferred axes of alignment. The three different shear viscosities that parameterise the different momentum diffusion processes relative to the director field are called Miesowicz coefficients [13].

distinguish between the closed string metric, in terms of which the supergravity solution is formulated and the open string metric, which is the metric that is seen by open strings ending on a D-brane in a B-field background [12].

In this paper we will argue that the correct variables to look at in order to define the gauge invariant operators in the dual field theory are the open string variables: the open string metric, the open string coupling and the Θ-parameter that defines the star product on the brane. The holographic stress tensor is therefore defined through the fluctuations of the open string metric. As we will see it is extremely important to realize this in order to be able to obtain a well defined field theory stress tensor from the supergravity dual. In fact it turns out that the temperature dependent part of the holographic stress tensor calculated with the help of the open string metric is precisely the same as the one in the usual commutative $\mathcal{N}=4$ theory. Our findings are in agreement with the arguments given in [14] and also in [15]

Since we define the stress tensor as the operator that is generated in the holographic supergravity dual by open string metric fluctuations it necessarily turns out to be symmetric. A consequence of this is that there are only two different Green-Kubo type formulas for the viscosity coefficients

$$\eta_1 = -\operatorname{Im}\frac{1}{\omega}\int dt\, d^3x\, e^{i\omega t}\theta(t)\langle[T_{yz}(t,\mathbf{x}),T_{yz}(0,0)]\rangle\,, \tag{1.3}$$

$$\eta_2 = -\operatorname{Im}\frac{1}{\omega}\int dt\, d^3x\, e^{i\omega t}\theta(t)\langle[T_{xy}(t,\mathbf{x}),T_{xy}(0,0)]\rangle\,, \tag{1.4}$$

that are not related to each other by either symmetry of the stress tensor or the SO(2) symmetry that rotates $(y \leftrightarrow z)$.

In the following we will compute both of the above stress tensor correlators and will also find the diffusion pole of the retarded correlator of the momentum density operator in y direction allowing for gradients in the x direction. We advance here that all three calculations end up in the universal value (1.1).

In the next section we present the supergravity background, establish our conventions and define an effective five dimensional theory by reduction on the S^5. In section three we compute the renormalised temperature dependent part of the holographic stress tensor. Section four is the core of this paper where we consider the various retarded Greens function outlined above and show that the shear viscosity takes the universal value (1.1) even in the anisotropic situation presented by the non-commutative theory. We summarise our conclusions in section five and outline possible interesting future investigations.

2. The background and dimensional reduction

In [10, 11] several geometries dual to non-commutative N=4 supersymmetric Yang-Mills theory where proposed. We shall concentrate on the one representing a finite temperature quantum field theory with non-commutative plane along spacelike directions (y, z). In the string frame the metric is

$$ds^2_{10,string} = \mathcal{H}^{-1/2}\left(-f dt^2 + dx^2 + h(dy^2+dz^2)\right) + \mathcal{H}^{1/2}(f^{-1}dr^2 + r^2 d\Omega_5^2)\,, \tag{2.1}$$

where

$$f = 1 - \frac{r_H^4}{r^4}\,; \quad h = \frac{1}{1+\theta^2\mathcal{H}^{-1}}\,; \quad \mathcal{H} = \frac{L^4}{r^4}\,. \tag{2.2}$$

The AdS/CFT dictionary sets $L^4 = 4\pi\hat{g}N\alpha'^2$ where $\hat{g} = g_{\rm YM}^2$. One easily sees that temperature and entropy density are independent of the non-commutativity parameter θ

$$T = \frac{r_H}{\pi L^2}\,; \qquad s = \frac{N^2\pi^2T^3}{2}\,. \tag{2.3}$$

In order to ease comparison with previous calculations in the literature we shall introduce new coordinates and definitions

$$u = \frac{r_H^2}{r^2}\,; \qquad u_T = \frac{r_H^2}{L^2} = (\pi T L)^2\,; \qquad a = \theta u_T\,, \tag{2.4}$$

in terms of which the full background acquires the following form, in the Einstein frame

$$ds^2_{10,E} = h^{-1/4}\left[\mathcal{H}^{-1/2}\left(-fdt^2+dx_1^2+h(dx_2^2+dx_3^3)\right)+\frac{L^2}{f}\frac{du^2}{4u^2}+L^2d\Omega_5^2\right], \tag{2.5}$$
$$f(u) = 1-u^2\,; \quad h(u) = \frac{u^2}{u^2+a^2}\,; \quad \mathcal{H}(u) = \frac{u^2}{u_T^2}\,,$$

and

$$\begin{aligned}
e^{2\phi} &= h\,,\\
H &= \frac{a}{u_T}(h\mathcal{H}^{-1})'dy\wedge dz\wedge dr\,,\\
F_{(3)} &= \frac{a}{u_T}(\mathcal{H}^{-1})'dt\wedge dx\wedge dr\,,\\
F_{(5)} &= h(\mathcal{H}^{-1})'dt\wedge dx\wedge dy\wedge dz\wedge dr+4L^4\omega_{(5)} =^* F_{(5)}\,.
\end{aligned} \tag{2.6}$$

In the rest of the paper we will work with an effective five dimensional theory that is defined by dimensional reduction along the S^5 sphere. Following [21] we define a five dimensional metric through

$$ds^2_{10,E} = e^{2\alpha\varphi}ds_5^2 + e^{2\beta\varphi}L^2d\Omega_5^2\,, \tag{2.7}$$

where $\alpha = \sqrt{5/3}/4$ and $\beta = -\sqrt{3/5}/4$ are chosen such that φ is a canonically normalised scalar[3] and ds_5^2 is the five-dimensional Einstein frame metric. The action of this effective five dimensional theory is given by[4]

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\left[\sqrt{-g_5}\left(\mathcal{R}-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}(\partial\varphi)^2-\frac{1}{2}e^{2\phi}(\partial\chi)^2-\frac{8}{L^2}e^{8\alpha\varphi}+e^{\frac{16}{5}\alpha\varphi}\mathcal{R}_5\right)\right. \tag{2.8}$$
$$\left.-\frac{1}{12}e^{\phi-4\alpha\varphi}F_{(3)}^2-\frac{1}{12}e^{-\phi-4\alpha\varphi}H^2-\frac{2}{L}\left(A_{(2)}\wedge dB-dA_{(2)}\wedge B\right)\right].$$

[3]On shell, the breathing mode φ is related to the dilaton ϕ. Since $ds^2_{10,E} = e^{-\phi/2}ds^2_{10,string}$, from (2.1), (2.5) and (2.7) it follows that $e^{2\beta\varphi} = e^{-\phi/2} = h^{-1/4}$, or $\varphi = \sqrt{\frac{5}{3}}\phi$.

[4]Our conventions are as follows

$$H = dB\,; \quad F_{(1)} = dA_{(0)}\;\;; \quad F_{(3)} = dA_{(2)} + A_{(0)}\wedge H\,; \quad F_{(5)} = dA_{(4)} + \frac{1}{2}A_{(2)}\wedge H - \frac{1}{2}B\wedge dA_{(2)}\,.$$

Here $\mathcal{R}_5$ denotes the curvature of the five-dimensional sphere of radius L, i.e. $\mathcal{R}_5 = \frac{20}{L^2}$ and $\chi = -A_{(0)}$. The five-dimensional gravitational coupling is given by $\kappa_5^2 = 4\pi^2 L^3/N^2$. The resulting equations of motion are given by

$$\begin{aligned}
\mathcal{R}_{\mu\nu} &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi+\frac{1}{2}e^{2\phi}\partial_\mu\chi\partial_\nu\chi+\frac{8}{3L^2}e^{8\alpha\varphi}g_{\mu\nu}-\frac{1}{3}e^{\frac{16\alpha}{5}\varphi}\mathcal{R}_5 g_{\mu\nu} \qquad (2.9)\\
&\quad+\frac{1}{4}e^{\phi-4\alpha\varphi}\left((F_{(3)})^2_{\mu\nu}-\frac{2}{9}(F_{(3)})^2 g_{\mu\nu}\right)+\frac{1}{4}e^{-\phi-4\alpha\varphi}\left(H^2_{\mu\nu}-\frac{2}{9}H^2 g_{\mu\nu}\right),\\
\Box\varphi &= \frac{64\alpha}{L^2}e^{8\alpha\varphi}-\frac{16}{5}\alpha e^{\frac{16\alpha}{5}\varphi}\mathcal{R}_5-\frac{\alpha}{3}e^{\phi-4\alpha\varphi}(F_{(3)})^2-\frac{\alpha}{3}e^{-\phi-4\alpha\varphi}H^2\,, \qquad (2.10)\\
\Box\phi &= (\partial\chi)^2 e^{2\phi}+\frac{1}{12}e^{\phi-4\alpha\varphi}(F_{(3)})^2-\frac{1}{12}e^{-\phi-4\alpha\varphi}H^2\,, \qquad (2.11)\\
d(e^{2\phi}*d\chi) &= -e^{\phi-4\alpha\varphi}H\wedge *F_{(3)}\,, \qquad (2.12)\\
d(e^{\phi-4\alpha\varphi}*F_{(3)}) &= \frac{4}{L}H\,, \qquad (2.13)\\
d(e^{-\phi-4\alpha\varphi}*H) &= -e^{\phi-4\alpha\varphi}d\chi\wedge *F_{(3)}-\frac{4}{L}F_{(3)}\,. \qquad (2.14)
\end{aligned}$$

3. The holographic stress tensor

The holographic stress tensor of the strongly coupled non-commutative $N = 4$ gauge theory has not been calculated until now in the literature. In fact holography itself is poorly understood in this background, mostly because the induced metric (2.7) at a fixed $r =$ const. scales anisotropically as one goes with $r \to \infty$ where it becomes degenerate. The authors of [14] observed however that the anisotropy in the dependence on r encodes just the anisotropic decoupling limit [10–12] for the metric components g_{yy} and g_{zz}. The same is true for the dependence of the dilaton on r. If one therefore asks what are the bulk fields that act as sources for the field theory operators it is necessary to use the open string variables defined in [12]

$$G_{\mu\nu} = g_{\mu\nu} - \left(Bg^{-1}B\right)_{\mu\nu}\,, \qquad (3.1)$$

$$\Theta^{\mu\nu} = 2\pi\left(\frac{1}{g+B}\right)^{\mu\nu}_A\,, \qquad (3.2)$$

$$G_s = g_s\left(\frac{\det G}{\det(g+B)}\right)^{\frac{1}{2}}\,, \qquad (3.3)$$

where the subscript A denotes antisymmetrisation. One can easily check that, for the background given in (2.1), the open string metric $G_{\mu\nu}$ is nothing but the usual (planar) AdS black hole, the open string coupling G_s is constant, and $\Theta^{32} = \theta$. In particular it follows that there is no anisotropy in the open string metric. Still, there is of course an anisotropy in terms of the open string variables because of the non vanishing Θ components which may lead to nontrivial physical effects (for example on the drag force of a moving quark [16]). We define the energy-momentum tensor in the strongly coupled non-commutative field theory as the operator that is sourced by the open string metric. Therefore we have to find out how the system responds to a variation of the open string metric keeping the non-commutativity Θ and the open string coupling G_s fixed.

However the five dimensional action is formulated in terms of $g^5_{\mu\nu}$, whereas the open string variables are written in terms of the closed string metric.

The solution to $\delta\Theta = 0$ gives the induced variations for the B-field. More explicitly:

$$\delta\Theta^{\mu\nu} = -2\pi \left[\left(\frac{1}{g+B} \right) (\delta g + \delta B) \left(\frac{1}{g+B} \right) \right]_A^{\mu\nu} = 0\,, \tag{3.4}$$

and solving for δB gives

$$\delta B = \frac{a}{u} \begin{pmatrix} 0 & 0 & -\delta g_{14} & \delta g_{13} & 0 \\ 0 & 0 & -\delta g_{24} & \delta g_{23} & 0 \\ \delta g_{14} & \delta g_{24} & 0 & \frac{u^2(\delta g_{33}+\delta g_{44})}{u^2-a^2} & \delta g_{45} \\ -\delta g_{13} & -\delta g_{23} & -\frac{u^2(\delta g_{33}+\delta g_{44})}{u^2-a^2} & 0 & -\delta g_{35} \\ 0 & 0 & -\delta g_{45} & \delta g_{35} & 0 \end{pmatrix} . \tag{3.5}$$

Similarly we find the induced variations on the dilaton. We set $\delta G_s = 0$, which is

$$\delta G_s = G_s \left[\delta\phi + \frac{1}{2}(G^{-1})^{\mu\nu}\delta G_{\mu\nu} - \frac{1}{2}\left(\frac{1}{g+B}\right)^{\mu\nu} (\delta g_{\mu\nu} + \delta B_{\mu\nu}) \right] = 0\,, \tag{3.6}$$

and find

$$\delta\phi = -\frac{H^{1/2}}{2h}\frac{a^2}{(u^2-a^2)}(\delta g_{33} + \delta g_{44})\,. \tag{3.7}$$

Notice that also the S^5 breathing mode φ is linked, on shell, with the dilaton (see footnote 3). Therefore we also have an induced variation of the form

$$\delta\varphi = \sqrt{\frac{5}{3}}\,\delta\phi\,. \tag{3.8}$$

It is important to notice here that we have written the variations of B and ϕ in terms of the variations of the closed string metric $g_{\mu\nu} = g^{10,string}_{\mu\nu}$. Now we need to relate these to the variations of the five dimensional Einstein metric g_5. From (2.7)

$$g^5_{\mu\nu} = e^{-2\alpha\varphi} g^{10,E}_{\mu\nu} = e^{-2\alpha\varphi-\phi/2} g^{10,string}_{\mu\nu} = e^{-4\phi/3} g_{\mu\nu}\,, \tag{3.9}$$

we obtain

$$\delta g^5_{\mu\nu} = h^{-2/3}\left(\delta g_{\mu\nu} - \frac{4}{3} g_{\mu\nu}\delta\phi\right)\,. \tag{3.10}$$

Following [22] we define a quasilocal stress tensor in the gravity theory as

$$\tau^{\rho\lambda}_G = \frac{2}{\sqrt{-\Gamma}}\frac{\delta S}{\delta\Gamma_{\rho\lambda}}\,, \tag{3.11}$$

where $\Gamma_{\mu\nu} = G_{\mu\nu} - \xi^r_\mu \xi^r_\nu$. such that ξ^r_μ is an outward looking normal vector to the hypersurface $r = const$ fulfilling $\xi^r_\mu \xi^r_\nu G^{\mu\nu} = 1$.

Since we work with the effective five-dimensional theory (2.8) we will first vary with respect to $g^5_{\mu\nu}$. We therefore also need the restriction of the five dimensional metric onto the hypersurface $r = const$. We will call this $\gamma_{\mu\nu}$, so $\gamma_{\mu\nu} = g^5_{\mu\nu} - \xi^{r,g}_\mu \xi^{r,g}_\nu$ and $\xi^{r,g}_\mu \xi^{r,g}_\nu g^{5,\mu\nu} = 1$.

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There are three contributions. The first is standard and comes from the variation of the purely gravitational Einstein-Hilbert and Gibbons-Hawking terms in the action

$$\tilde{\tau}_g^{\rho\lambda} = \frac{1}{\kappa_5^2}\left(K^{\rho\lambda} - \gamma^{\rho\lambda}K_c^c\right) , \tag{3.12}$$

where $K_{\mu\nu}$ is the extrinsic curvature of the metric that is induced at $r = const$ by the fivedimensional metric $g^5_{\mu\nu}$. The other contributions come from the induced variations on the B-field and the dilaton and are given by

$$\begin{aligned}
\tilde{\tau}_B^{\rho\lambda} &= -\frac{1}{\kappa_5^2}\xi_\tau^r\left(e^{-\phi-4\alpha\varphi}H^{\tau\sigma\kappa} + \frac{1}{L}\epsilon^{\tau\mu\nu\sigma\kappa}A_{\mu\nu}\right)\frac{\delta B_{\sigma\kappa}}{\delta\gamma_{\rho\lambda}} , && (3.13)\\
\tilde{\tau}_\phi^{\rho\lambda} &= -\frac{1}{\kappa_5^2}\xi_\tau^r\partial^\tau\phi\frac{\delta\phi}{\delta\gamma_{\rho\lambda}} , && (3.14)\\
\tilde{\tau}_\varphi^{\rho\lambda} &= -\frac{1}{\kappa_5^2}\xi_\tau^r\partial^\tau\varphi\frac{\delta\varphi}{\delta\gamma_{\rho\lambda}} . && (3.15)
\end{aligned}$$

All the $\tilde{\tau}_{\mu\nu}$'s have been defined multiplied with a factor of $\frac{2}{\sqrt{-\gamma^5}}$. Now we need to convert $\tilde{\tau}_g^{\mu\nu}$ into $\tau_G^{\mu\nu}$ according to[5]

$$\tau_I^{\mu\nu} := \frac{\sqrt{-\gamma}}{\sqrt{-\Gamma}}\sum_{\lambda\leq\rho}\tilde{\tau}_I^{\rho\lambda}\frac{\delta g^5_{\rho\lambda}}{\delta G_{\mu\nu}} \qquad ; \qquad I \in \{g, B, \phi, \varphi\} . \tag{3.16}$$

The total stress tensor is now simply the sum of the different contributions

$$\tau_{\text{total}}^{\mu\nu} = (\tau_G^{\mu\nu} + \tau_B^{\mu\nu} + \tau_\phi^{\mu\nu} + \tau_\varphi^{\mu\nu}) . \tag{3.17}$$

Finally we define the vacuum expectation value of the stress tensor in the field theory as

$$\langle T^{ab}\rangle = \lim_{r\to\infty}\sqrt{-\Gamma}\xi_\rho^a\xi_\lambda^b\tau_{G,\text{reg}}^{\rho\lambda} . \tag{3.18}$$

In the last step a suitable regularization procedure has to be applied on $\tau_{G,\text{total}}$. In principle one would need to construct local counterterms on the boundary to cancel the divergences as in [24]. In the case of the non-commutative background these counterterms have not yet been constructed in the literature. Therefore we employ the somewhat simpler method of subtracting the gravity stress tensor of a reference spacetime before taking the limit $r \to \infty$ [22]. As reference spacetime we take the zero temperature solution. This is sufficient for our purposes since we are only interested in the temperature dependent part of the stress tensor. We find

$$\langle T^{ab}\rangle = \frac{\pi^2 T^4 N^2}{8}\text{diag}(3, 1, 1, 1) , \tag{3.19}$$

which is exactly the same result as for the commutative theory. It is worth emphasising the contrast between this result and the one found in [23] where the stress-energy tensor of an asymptotically flat background produced by a $D1 - D3$ bound system was computed and found to be anisotropic. In particular taking the naive decoupling limit in this solution leads to negative pressures. It would be interesting to find an interpretation for this mismatch.

[5]We should actually compute $\delta\gamma^5_{\rho\lambda}/\delta\Gamma_{\mu\nu}$, but there is a projection implicitly in the definition of τ so that it is OK to use the full metrics here.

4. Fluctuations

The system of equations (2.9)-(2.14) involves a large number of fields. Therefore, looking for decoupled fluctuations is now a more involved task, and calls for symmetry analysis. In the search for dispersion relations, that show up as poles in retarded correlators, it is customary to assume a plane wave like perturbation of the form $\Phi(u,k) = \phi(u)e^{ikx}$. For $k^\mu = (-\omega, \vec{k})$ this ansatz leaves a little group $O(2)$ of rotations in the transverse plane. There is another $O(2)$ remnant of the isotropy group that is broken by the background as soon as the anticommutativity parameter $a \neq 0$ is switched on. When $\vec{k}$ has a component along the noncommutative plane the symmetry group is completely broken. Perturbations involve typically all the fields and become extremely tedious to handle. The easiest situations occurs when $\vec{k} = (k, 0, 0)$, since in this case there is an overall $O(2)$ symmetry group and the usual tensor decomposition applies. In this paper, only such a family of plane waves will be considered. We will examine in detail, only two channels which directly measure the shear viscosity: the scalar channel ($O(2)$ helicity $h = 2$), and one of the vector channels ($h = 1$). In this section $g_{\mu\nu}$ will always stand for the five-dimensional Einstein frame metric.

4.1 Scalar channel

When the polarization of the metric fluctuation lives inside the non-commutative (y, z)-plane , δg_{yz} rotates with $O(2)$ as a tensor of spin 2 and decouples from the rest. For such a polarization we expect to find no poles on the retarded correlator. Therefore, the usual alternative is to use Kubo formula, and for this, just a time dependent fluctuations is enough $\delta g^y{}_z = e^{-i\omega t}\rho_\omega(u)$. The equation that $\rho_\omega(u)$ satisfies is the usual one for a minimally coupled scalar

$$\rho_\omega'' - \frac{1+u^2}{uf}\rho_\omega' + \frac{\mathfrak{w}^2}{uf^2}\rho_\omega = 0\,, \tag{4.1}$$

whose solution, as an expansion in powers of $\mathfrak{w} = \frac{\omega}{2\pi T}$ is the usual one

$$\rho_\omega(u) = (1-u^2)^{-\frac{i}{2}\mathfrak{w}}(1 + \mathcal{O}(\mathfrak{w}^2) + \ldots)\,. \tag{4.2}$$

The normalisation $\rho_\omega(0) = 1$ is in agreement with the fact that the five-dimensional perturbation and the open-string perturbations are related by $\delta g^y{}_z = \delta G^y{}_z$. In a sense, this anticipates the expected fact that in this channel no signal of the non-commutative effects will show up. It can be further checked by computing the boundary gravitational action. A Fourier synthesis like $\delta g^y{}_z(t,u) = \int \frac{d\omega}{2\pi} e^{-i\omega t} f(\omega)\rho_\omega(u)$ may be plugged into the bare action

$$S = S_5 + S_{\rm GH}\,, \tag{4.3}$$

where $S_{\rm GH} = \frac{1}{8\pi G_5}\int_{u=\epsilon} d^4x K$ stands for the usual Gibbons-Hawking term. After some algebra, the boundary action is found to be

$$S = \int d^3x \frac{d\omega}{2\pi}\, f(\omega)f(-\omega)\mathcal{F}(u)\Big|_{u=\epsilon}^{u=1}\,, \tag{4.4}$$

where the flux is given by

$$\mathcal{F}(u) = -\frac{N^2}{8\pi^2 L^3}\left(\frac{u_T^2((3u^2-5)a^2+u^2(u^2-3)}{Lu^2(a^2+u^2)} - i\mathfrak{w}\frac{u_T^2}{L}+\ldots\right). \tag{4.5}$$

The divergence at $u = 0$ calls for boundary counterterms to be added to the bare action (4.3). However it only shows up in the real part! Using the holographic recipe for calculating retarded Greens functions [18] and the Green-Kubo formula (1.3) we find

$$\eta_1 = \lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}(-2\mathcal{F}(u=0)) = \frac{N^2\pi T^3}{8}\,, \tag{4.6}$$

which leads to the universal value $\eta/s = 1/4\pi$ for the shear viscosity to entropy ratio.

4.2 Vector channel: dispersion relations

Let us now choose $\delta g_x{}^z$. This perturbation has an index along the propagation axis x and another one, z, along the non-commutative plane.[6] The irreducible set of fluctuations involves six fields components, all transforming as vectors of $O(2)$. Setting $\delta g_t{}^z = e^{-i(\omega t+kx)}\rho_t$, $\delta g_x{}^z = e^{-i(\omega t+kx)}\rho_x$, $\delta A_{tz} = e^{-i(\omega t+kx)}u_T\alpha_t$, $\delta A_{xz} = e^{-i(\omega t+kx)}u_T\alpha_z$ and $\delta B_{ty} = e^{-i(\omega t+kx)}u_T\beta_t$, $\delta B_{xy} = e^{-i(\omega t+kx)}u_T\beta_x$, the six fields $\rho_x, \rho_t, \alpha_t, \alpha_z, \beta_t$ and β_x satisfy a system of six coupled second order ordinary differential equations, plus three constraints. The system is not overdetermined and one can find easily three linear differential relations that automatically vanish. As explained in [19], the natural variables to look for, are combinations which are invariant under the residual coordinate gauge transformations.

$$Z = \mathfrak{q}\rho_t + \mathfrak{w}\rho_x\,, \tag{4.7}$$
$$V = \mathfrak{q}\alpha_t + \mathfrak{w}\alpha_x\,, \tag{4.8}$$
$$W = \mathfrak{q}\beta_t + \mathfrak{w}\beta_x\,. \tag{4.9}$$

This set of variables collapses into a system of 3 coupled differential equations. Further decoupling occurs if one defines the new set $(Z, V, W) \to (P, V, Q)$ with

$$P = Z - \frac{a}{h}W \quad ; \quad Q = W + \frac{ah}{u^2}Z\,. \tag{4.10}$$

Then we find a coupled system for (V, Q)

$$Q'' - \frac{\mathfrak{q}^2 f^2 + (3u^2-1)\mathfrak{w}^2}{Ufa(\mathfrak{w}^2 - f\mathfrak{q}^2)}Q' + \frac{u(\mathfrak{w}^2 - f\mathfrak{q}^2) - 4f}{u^2 f^2}Q - \frac{4\mathfrak{q}\mathfrak{w}}{f(\mathfrak{w}^2 - f\mathfrak{q}^2)}V = 0\,, \tag{4.11}$$
$$V'' - \frac{\mathfrak{q}^2 f^2 + (3u^2-1)\mathfrak{w}^2}{Ufa(\mathfrak{w}^2 - f\mathfrak{q}^2)}V' + \frac{u(\mathfrak{w}^2 - f\mathfrak{q}^2) - 4f}{u^2 f^2}V - \frac{4\mathfrak{q}\mathfrak{w}}{f(\mathfrak{w}^2 - f\mathfrak{q}^2)}Q = 0\,, \tag{4.12}$$

[6] This choice of index rising simplifies the equations, and affects the normalisation of the correlator, but does not modify the position of the searched poles.

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and a decoupled equation for P

$$P''-h\left(\frac{((u^2+1)\mathfrak{w}^2-f^2\mathfrak{q}^2)}{Ufa(\mathfrak{w}^2-f\mathfrak{q}^2)}+a^2\frac{3\mathfrak{q}^2f^2+(5u^2-3)\mathfrak{w}^2)}{u^3f(\mathfrak{w}^2-f\mathfrak{q}^2)}\right)P'+h^2\left(\frac{\mathfrak{w}^2-f\mathfrak{q}^2}{Ufa^2}\right.$$
$$+a^2\frac{2uf^2\mathfrak{q}^4+4f(2u^4-4u^2-u\mathfrak{w}^2+2)\mathfrak{q}^2+2\mathfrak{w}^2(-2u^4+6u^2+\mathfrak{w}^2u-4)}{u^4f^2(\mathfrak{w}^2-f\mathfrak{q}^2)}$$
$$\left.+a^4\frac{f^2q^4-2f\mathfrak{w}^2\mathfrak{q}^2+\mathfrak{w}^2(\mathfrak{w}^2-4Ufa)}{u^5f^2(\mathfrak{w}^2-f\mathfrak{q}^2)}\right)P=0\,.$$

The system (4.11) and (4.12) is independent of a. The natural combination to consider as a gravitational perturbation is P, and we see that the equation for it is explicitly dependent on a, signalling a possible influence of this parameter in the final solution. However, as usual, we shall try a perturbative solution in λ

$$P(u)=f(u)^{-i\mathfrak{w}/2}\left(P_0(u)+\lambda P_1(u)+\ldots\right)\,, \tag{4.13}$$

with $\mathfrak{w}\to\lambda(-i\Gamma q^2), \mathfrak{q}\to\lambda\mathfrak{q}$, and normalised as $P_0(1)=1, P_1(1)=0$. One readily obtains

$$P(u)=f(u)^{-i\mathfrak{w}/2}\frac{1}{h(u)(a^2+1)}\left(1+i\frac{\mathfrak{q}^2}{2\mathfrak{w}}f(u)+\mathcal{O}(\mathfrak{w}^2,\mathfrak{q}^4,\mathfrak{w}\mathfrak{q}^2)\right)\,. \tag{4.14}$$

It is helpful now to remember the induced variations on the B-field in (3.5). Demanding as before $\delta\Theta=0$ we find that in the new variables this is just $Q=0$. Using this and plugging the solution for W back into the definition of P we find $P=Z/h$. If we want to know the dispersion pole we need to set $Z=hP|_{u=1}=0$. Notice that the overall normalisation of the solution for P or Z is irrelevant here! We observe therefore that the hydrodynamic pole sits at the same position as in the commutative case [19].

$$\mathfrak{w}=-i\frac{\mathfrak{q}^2}{2}\,. \tag{4.15}$$

This is however not yet enough to infer that the shear viscosity is indeed given by its universal value since the diffusion constant for momentum diffusion is actually given by $\frac{\eta}{\epsilon+p}$. So far we have only learnt that the diffusion constant is $D=\frac{1}{4\pi T}$. Now it is important to know the energy momentum tensor in equilibrium. As we have shown explicitly it is given by the same expression as in the commutative case and obeys $\epsilon+p=Ts$, from which we can infer now the value of the shear viscosity as

$$\frac{\eta_2}{s}=\frac{1}{4\pi}\,. \tag{4.16}$$

4.3 Vector channel: Kubo formula

In the previous section we have computed the pole in the retarded correlation function of the gauge invariant variable Z. Since Z contains $\delta g^t{}_z$ and the latter is the source for the momentum density $\mathcal{P}_z$ in z-direction what we really have computed is the diffusion constant for the process of momentum pointing along the non-commutative z-direction and diffusing into the commutative x-direction. By the remnant SO(2) symmetry this is the same as the diffusion constant for the process of the y-momentum diffusing into x-direction. On the other hand we have seen already, that viscous flow taking place in the non-commutative plane along is governed by the universality of the shear viscosity. This leaves us to determine the diffusion constant for the process of momentum density along the commutative direction p_y diffusing into a non-commutative direction x. To this end we will employ the Kubo formula [20]

$$\eta_3 = -\lim_{\omega\to 0} \frac{\mathrm{Im}\, G_{xy,yx}(\omega,0)}{\omega}\,, \tag{4.17}$$

where $G_{xy,yx} = G_{xy,xy}$ is the retarded Greens function of T_{xy}. In fact the symmetry of the stress tensor defined through the variation of the open string metric implies immediately that $\eta_3 = \eta_2$, so we should recover (4.16). Nevertheless, it is an interesting exercise to recover the result of the previous section from the Kubo formula. Consider the purely time-dependent set of perturbations

$$\begin{aligned} \delta g^x{}_z &= \int \frac{d\omega}{2\pi} e^{-i\omega t}\Phi(\omega)\rho_\omega(u)\,,\ \delta A_{tz} \\ &= u_T \int \frac{d\omega}{2\pi} e^{-i\omega t}\Phi(\omega)\alpha_\omega(u)\,,\ \delta B_{xy} \\ &= u_T \int \frac{d\omega}{2\pi} e^{-i\omega t}\Phi(\omega)\beta_\omega(u)\,. \end{aligned} \tag{4.18}$$

One readily finds that α_ω obeys a first order equation

$$\alpha'_\omega = -\frac{2a}{u^3}\rho_\omega - \frac{2}{u}\beta_\omega. \tag{4.19}$$

Inserting this into the equations for ρ and β, they can be casted as follows

$$\rho''_\omega + \left(\frac{2h}{u} - \frac{3-u^2}{uf}\right)\rho'_\omega + \frac{\mathfrak{w}^2}{uf^2}\rho_\omega - \frac{2ah}{u}\left(\frac{2}{uf}\beta_\omega - \beta'_\omega\right) = 0\,, \tag{4.20}$$

$$\beta''_\omega + \left(\frac{2h}{u} - \frac{1+u^2}{uf}\right)\beta'_\omega + \left(\frac{\mathfrak{w}^2}{uf^2} - \frac{4h}{u^2 f}\right)\beta_\omega - \frac{2ah}{u^3}\rho'_\omega = 0\,. \tag{4.21}$$

We expect four independent solutions to this system, two of which will be incoming at the horizon. Parametrising the general solution by two constants, which we take to be the boundary values $\rho(0) = \rho_0$ and $\beta(0) = \beta_0$ we find, up to order $\mathcal{O}(\mathfrak{w}^2)$

$$\rho(u) = f(u)^{-i\mathfrak{w}/2}\frac{\rho_0(a^2+u^2) - \beta_0 u^2 a}{a^2+u^2}\left(1 - \frac{i\mathfrak{w}}{2}\frac{\rho_0 u^2 a^2}{\rho_0(a^2+u^2) - \beta_0 u^2 a}\right)\,, \tag{4.22}$$

$$\beta(u) = f(u)^{-i\mathfrak{w}/2}\frac{\beta_0 a^2}{a^2+u^2}\left(1 - \frac{i\mathfrak{w}}{2}\frac{\rho_0 u^2}{\beta_0 a}\right)\,, \tag{4.23}$$

and from these, $\alpha(u)$ can be obtained integrating (4.19)

$$\alpha(u) = f(u)^{-i\mathfrak{w}/2}\frac{\rho_0 a f(u)}{u^2}\left(1 + \frac{i\mathfrak{w}}{2}C_\alpha\frac{u^2}{\rho_0 a f(u)}\right)\,. \tag{4.24}$$

Now, in order to proceed, we must compute the boundary action. As usual, on shell, the regulated action $S = S_{\rm Bulk} + S_{\rm GH} + S_{\rm ct}$ can be expressed in terms of boundary data

$$S = \int d^3x \frac{d\omega}{2\pi}\, \Phi(\vec{x},\omega) \Big(\mathcal{F}_{\rm Bulk}(\omega,u)|_{u=\epsilon}^{u=1} + \mathcal{F}_{\rm GH}(\omega,\epsilon) + \mathcal{F}_{\rm ct}(\omega,\epsilon) \Big) \Phi(\vec{x},\omega)\,. \tag{4.25}$$

We don't know the structure of counterterms. However, one can prove that the imaginary part of $\mathcal{F}_{\rm Bulk}(\omega,u) + \mathcal{F}_{\rm GH}(\omega,u)$ is already a conserved flux, and hence independent of u.

$$\mathrm{Im}\Big(\mathcal{F}_{\rm Bulk}(\omega,u) + \mathcal{F}_{\rm GH}(\omega,u) \Big) = -\mathfrak{w} \frac{u_T^2}{L} \frac{N^2}{8\pi^2 L^3} \left(\rho_0^2 + (\beta_0^2 + \rho_0^2) a^2 - 2\beta_0 \rho_0 a \right)\,. \tag{4.26}$$

This is all we need in order to compute the shear viscosity, and conforms with the expectation that the hydrodynamic transport coefficients should not depend on the UV details of the theory.

Comparing with (4.5), we see that the resulting value of the shear viscosity depends on the correct choice of boundary conditions. Each one selects a corresponding quantum operator in the boundary theory.

It is very interesting to compute the result corresponding to the operator O_5 that couples to the five dimensional closed string bulk metric g_5. This can be done by simply setting $\rho_0 = 1, \beta_0 = 0$. Plugging this into (4.26) and (4.17) we would end up with

$$\mathrm{Im}\, G_{O_5,O_5} = -\omega \frac{s}{4\pi}(1+a^2)\,. \tag{4.27}$$

Using this would lead to a non-universal value for the shear viscosity and moreover would be in conflict with the results in the previous section.

Let us consider now the correct boundary stress tensor defined as the operator that couples to the open string metric. Using (3.5) it is easy to see that

$$\beta = -\frac{a}{u^2}\rho\,. \tag{4.28}$$

solves the constraint for varying only the open string metric. For the solutions given in (4.22) and (4.23), the choice $\rho_0 = 0, \beta_0 = 1/a$ fulfils this condition. Using furthermore the definition of the open string metric (3.1) it is not difficult to see that this corresponds precisely to an open string perturbation that obeys $\delta G^x{}_z = 1$ at the boundary. Using this solution we find now for the imaginary part of the retarded correlator

$$\mathrm{Im}\, G(\omega, 0) = -\omega \frac{s}{4\pi}\,. \tag{4.29}$$

as expected.

5. Conclusions and discussion

We have calculated the shear viscosity to entropy ratio in the holographic dual to a non-local non-commutative field theory. Although this theory is intrinsically anisotropic it turned out that the shear viscosities satisfy the conjectured bound exactly. We think this a rather remarkable result and collects further strong evidence in favour of the conjecture.

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Although we were able to solve some important problems in this particular holographic theory there are many interesting questions that could be addressed in future work.

One puzzling question is related to the membrane paradigm. It has been shown that hydrodynamic behaviour of black holes also arises in a purely gravitational perspective [25]. Explicit gravitational counterparts of shear modes have been constructed in the membrane paradigm approach. In the non-commutative theory under consideration we emphasized the importance of the distinction between the open and closed string metric. From the pure gravity perspective however there is no reason a priori to consider the particular combination of fields defining the open string metric. From a bulk gravity perspective it is far more natural to consider the closed string metric. Using therefore the membrane paradigm approach would suggest to consider closed string metric perturbations and it is far from clear if the hydrodynamic shear modes defined within the membrane paradigm using the bulk metric will give rise to the universal result (1.1). This points to a possible discrepancy between the membrane paradigm and holography!

Another aspect concerns the holographic renormalization of the non-commutative theory. We have successfully implemented a simple subtraction scheme to compute a renormalised stress tensor. A much more powerful approach of holographic renormalization is to construct local covariant counterterms on the boundary. Such a program looks extremely difficult to realize if one considers the closed string variables, mostly because of the anisotropic behaviour in the holographic coordinate u the non-commutative and commutative directions. We have pointed out that this anisotropy in the scaling with r is somewhat fiducial since the correct variables to consider are the open string variables in which the metric is perfectly isotropic. This suggests that the holographic renormalization program of the non-commutative theory should be based on the the open string variables!

A new difficulty arises when one switches on momentum q in the non-commutative direction. It turns out that the differential equations for the linear fluctuations of the bulk fields are not any more of Fuchsian type but acquire essential singularities at the boundary. Moreover, many more fluctuations mix with each other than for the fluctuations considered in this paper. Therefore this presents as much a technical difficulty due to the high level of mixing as a conceptual one due to the presence of the essential singularities at the boundary.

Finally we want to point out that the techniques developed in this paper should be sufficient to calculated the stress tensor and the stress tensor correlators at zero momentum also in other non-commutative backgrounds such as the non-commutative open string theory [26]. We hope to make progress on these questions in future work.

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